Access Algebra: *Effective Strategies for Promoting Informal Mathematics Learning*

Prepared for

OREGON MUSEUM OF SCIENCE AND INDUSTRY

by
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with the generous support of

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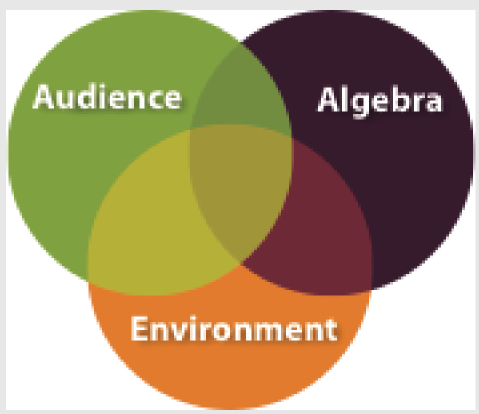

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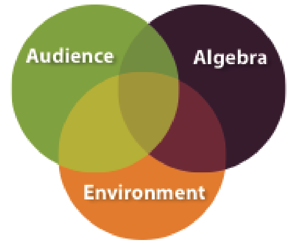
Executive Summary



The Oregon Museum of Science and Industry (OMSI) received a five-year grant from the National Science Foundation (NSF) to develop the *Access Algebra: Effective Strategies for Promoting Informal Mathematics Learning* project. *Access Algebra* will include a traveling exhibition and a comprehensive professional development program for educational staff at host museums. *Access Algebra* will target middle-school age youth and engage visitors in design activities that promote creativity and innovation and build mathematics literacy by taking a non-traditional, experiential approach to mathematics learning.

Oregon State University (OSU) faculty from the College of Education and the College of Science with expertise in mathematics teaching, administrative experience at K–20 levels and within the Oregon Department of Education, and in free-choice learning settings and research are collaborating on research and development for the exhibit prototypes and professional development components of the project.

This report is intended to be a snapshot of relevant empirical research to assist OMSI staff in the development of the *Access Algebra* exhibition and activities. The following sections are included in this report:

- *Audience:* adolescent behavior and adolescent development research, as well as research focused on families
- *Algebra:* research focused on mathematical thinking and more specifically on algebraic thinking with a focus on efforts involving adolescents and families
- *Environment:* research focused on free-choice learning environments and activities
- *Recommendations:* strategies for the three areas (audience, algebraic thinking, and effective learning strategies in free-choice learning environments)
- *References and Annotated Bibliography:* includes all references used in preparing this report and specific annotated citations and “must read” documents
- *Appendix:* provides a list of other mathematics exhibitions at museums across the country.

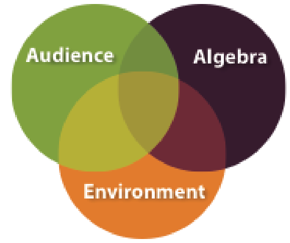
Audience: Adolescent Behavior and Development Research and Research on Families

Research on adolescent behavior and development suggests that identity formation, autonomy, and relationships with peers and to some degree, adults are primary motivators for children of this age. As adolescents mature they grow increasingly goal directed and interested in lifelong learning, particularly if they are fortunate to live in supportive homes where appropriate challenges are encouraged and supported. Effective learning experiences for adolescents focus on tapping into intrinsic motivation, allowing for concentration and goal-directness, and result in real products. Optimal conditions for learning include a balance between play and work, often referred to as serious play. Activities that are open-ended enough for multiple approaches and solutions and challenging, yet not too overwhelming, fit into this category. Peer to peer collaboration are successful when children take turns and build on the content of their conversations that enhances the problem-solving abilities of the group as a whole. In a more naturalistic setting, peer groups composed of an adolescent and a younger child, both benefited physically, intellectually, and socially from their interactions with each other.

Algebra: Algebraic Thinking

Algebraic thinking is a vehicle for exploring the world, by observing its regularities, relationships, and patterns. Children of all ages (elementary, middle, and high school) can use thinking to investigate phenomena and relationships, make predictions/conjectures, arrive at creative, open-ended solutions, and generally appreciate the elegance and efficiency of mathematics. However, research shows that children often have difficulty recognizing mathematics in everyday activity if it is considered too easy, there is no obvious problem to solve, or the solution is intuitive rather than formal. There is also a tendency for children to see mathematics as striving for the “one right answer.” However, when the environment affords, and facilitators foster a culture of figuring out, in which no one feels dumb and there is respect for diverse ideas, this notion of mathematics can change. This culture is best achieved when there is optimal challenge and learners are empowered through collaboration and talk.

Multiple entry points allow individuals to decide for themselves how to use the available resources for their own learning and also highlights the need for facilitators to be aware of multiple zones of proximal development at work in the same space. Facilitators also need to be aware of perceived group status in terms of comfort with and understanding of algebraic ideas since status can affect learning in groups. An important finding from two mathematics exhibitions in museums was how often they evoked powerful emotional responses, both negative and positive.



Environment: Research Focused on Free-Choice Learning Environments and Activities

Somewhat novel settings, like museums, are stimulating and exciting, and visitors intuitively seek them out because of their moderate yet "safe" levels of novelty. Good design is appealing, draws learners in, engages all senses, and compels them to investigate the topic at hand. Free-choice learning is optimized when there are opportunities for cultural, social, and cross-cultural interaction and spaces are comfortable and inviting, allowing people to congregate, engage, and learn together. Spaces youth can call their own are important and larger spaces that allow for games, group projects, and cultural festivals are also important for cultural-social interaction. Effective activities in free-choice learning settings start with the personal interests of visitors, afford collaboration and talk, and often include elements of fantasy and imagination. Activities that are purposeful, and offer choice and ownership are also important. Embedding activities into relevant everyday life, using narrative and emotional elements can also foster interest and engage learners of all ages.

Recommendations

The key to an effective exhibition is to maximize the overlap of audience needs, the richness of mathematical/algebraic thinking and doing with effective learning strategies and design approaches as much as possible. The following recommendations emerge from initial research synthesis and target this overlap.

- Create exhibition spaces that are comfortable and inviting, draw learners in, engage all of their senses, provide opportunities for cultural, social, and cross-cultural interaction.

- Design exhibit spaces that acknowledge and ameliorate negative emotions and build on positive emotions.

- Provide many and varied opportunities for visitors to personally identify with the exhibition through images and the "natural language" of people in the exhibition who look like and talk like "cool" adolescents or young adults; (e.g., the *Numb3rs* TV show).

- Games, puzzles, stories, and group projects that result in real products are potentially important avenues since they afford opportunities for active engagement in algebraic thinking through collaboration, conversation, use of imagination and fantasy, and narrative approaches, and, if designed well, include many of the elements above.

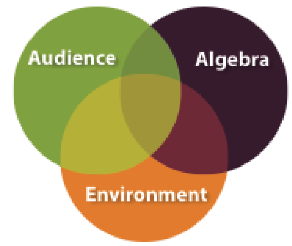
- There may be ways to have activities incorporate feedback from previous visitor groups, extend stories, challenges over time, or continue to interact remotely using technology.

- The development team should explore innovative ways of using technology to extend the experience beyond the walls of the museum and as advance organizers accessible prior to the exhibition experience (e.g., IPod downloading, computer chats/online discourse communities, MySpace).

- Important factors to include in the professional development component of this project include:

    - Adolescents learn best when supported and challenged individually at their own level of understanding so facilitators need to be aware of multiple zones of proximal development at work in the same space at the same activity.
    - Design activities with multiple entry points which allow learners themselves to select the activity, gravitate towards their own optimal challenge, and decide for themselves how to best use the available resources for their own learning.
    - Make facilitators aware of perceived group status in terms of comfort with and understanding of algebraic thinking since such status can affect learning in groups even at early ages; they should be trained as facilitators not “teachers”; their role is to ask questions not to provide the “right answer”; train facilitators to foster a culture of figuring out (algebraic thinking) in which no one feels dumb and there is respect for diverse ideas, multiple solutions, multiple strategies, and multiple entry points.

Introduction



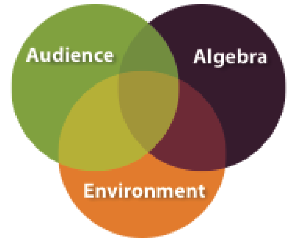
The Oregon Museum of Science and Industry (OMSI) has received a five-year grant from the National Science Foundation (NSF) to develop the *Access Algebra: Effective Strategies for Promoting Informal Mathematics Learning* project. *Access Algebra* will include a traveling exhibition to be hosted by other museums and a substantial professional development program for educational staff at the host museums. *Access Algebra* will target middle-school age youth and their families. The goal of the exhibit and the activities is to promote creativity, innovation, and enhance *algebraic thinking* by taking an experiential approach to learning mathematics.

Faculty members from the College of Education and the College of Science at Oregon State University (OSU) are collaborating with OMSI staff and others on the development of the *Access Algebra* project. The OSU faculty members who developed this report have extensive experience and expertise in mathematics education and free-choice learning settings such as museums.

This report is intended as a snapshot of the relevant empirical research and is designed to assist OMSI staff in the development of the *Access Algebra* exhibition and activities. The following databases available through the OSU Library System were used to compile this literature review: Education Resources Information Center (ERIC), Academic Search Primer, Psychological and Behavior Sciences Collection, Education Research Complete, Professional Development Collection, and Google Scholar Social Sciences. Keywords included free-choice, informal, everyday, out-of-school, learning, education, mathematics, algebra, adolescents, children, family/families, 10- to 14-year-olds, and underrepresented groups.

The report is organized around the following sections identified as critical to the development of *Access Algebra*:

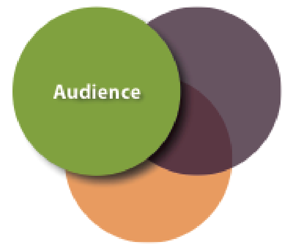
- *Executive Summary:* brief overview of the literature review and its findings
- *Literature Synthesis*
  - *Audience:* adolescent behavior and adolescent development research, as well as research focused on families
  - *Algebra:* research focused on mathematical thinking and more specifically on algebraic thinking with a focus on efforts involving adolescents and families
  - *Environment:* research focused on free-choice learning environments and activities



- *Recommendations:* strategies for the three areas (audience, algebraic thinking, and effective learning strategies in free-choice learning environments)
- *References:* complete set of citations from the *Literature Synthesis*
- *Annotated Bibliography:* annotated citations of the most relevant literature and identification of "must read" articles/reports
- *Appendix:* list of mathematics exhibitions developed nationally (some are no longer available but provide a sense of what has been accomplished in this area)

Literature Synthesis: Audience

Adolescent Behavior and Development Research and Research on Families



Several themes emerge from research on adolescent behavior and development. Identity formation, autonomy, and relationships with peers are primary motivators for children of this age. Research into adolescence generally, and well-being in these years specifically, suggests that it is a period involving complicated interactions as social and personal characteristics evolve and change. However, though adolescents experience a complexity of behaviors and experiences, there is also evidence that it may not be as tumultuous a time as previously thought. Measures of internal locus of control and self-esteem steadily increase with age during this period, while positive affective states decrease. Self-worth decreases until 15 or 16 years of age and then increases (Csikszentmihalyi and Schneider, 2001). Race/ethnicity, socioeconomic background, and gender are also complicating factors that can increase stress and inhibit coping strategies.

This is also an interesting time cognitively. There is a great deal of activity in the adolescent brain and it is not all related to fluctuating hormones. Research suggests that areas of the brain identified as involved with executive function and social cognition undergo major reconstruction during this period. The brain also is exceedingly plastic during this time which is thought to potentially expand repertoires of thinking that enable adolescents to solve increasingly adult problems (Luna, 2004). And although adolescents may seem very adult-like, researchers also caution that they still think differently than adults. In addition adolescents are at different stages of developing abstract thinking and so it is important to recognize that there is likely great variation between and among individual youth in peer groups.

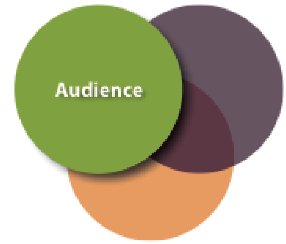
As adolescents become adults they grow increasingly goal directed and interested in lifelong learning, particularly if they are fortunate to live in homes where appropriate challenges are encouraged and supported. Fortunately, there is evidence that youth who may not benefit from such home lives can be supported effectively in after-school or community-based settings, as long as there are caring and supportive adults around. Effective learning experiences for adolescents focus on tapping into intrinsic motivation, foster concentration and goal-directness, and result in real products (Csikszentmihalyi and Schneider, 2001; Rathunde, 2001). Optimal conditions for learning include a balance between play and work, what is often referred to as serious play. Activities that are open-ended enough for multiple approaches and solutions and challenging, yet not too overwhelming as to limit success, fit into this category. This playful-seriousness fosters undivided attention and engagement and can result in flow experiences.

Socio-cultural influences, particularly among and between peers, are paramount and can have positive or negative effects on leisure learning. Research in classrooms demonstrates that, even at early ages children working in peer groups perceive status within and between groups and the perception of status in terms of academic achievement among individuals in the group affects the learning within the group as a whole. Individual members can be positively or negatively influenced by these group dynamics far into the future (Matthews & Kesner, 2003; Nesdale & Flesser, 2001).

In small groups or between pairs of children, peer-to-peer collaboration can be successful when the children take turns and build on the content in the conversation. Such collaborative talk between children seems to enhance the problem-solving abilities of the group as a whole (Denner & Werner, 2007; Teasley, 1995). In a more natural democratic school setting, Gray & Feldman (2004) found that peer groups composed of an adolescent and a younger child benefited in their interactions with each other. Because classes were not compulsory, when an adolescent interacted with a younger child they did so out of mutual interest and shared play style, personal attraction, and complementary desires to nurture and be nurtured. Both the older and younger child benefited by the interaction physically, intellectually, and socially with adolescents fostering a zone of proximal development in which the younger child benefited, while the younger child required the older child to practice leadership skills and make their thinking explicit and creative (Gray & Feldman, 2004).

Parents and guardians are still quite influential in terms of adolescent learning and leisure activities, especially in the case of the younger adolescent. Since parents, and especially mothers, are often still making the decision about what activities their children can choose, they play a critical role in whether youth participate in certain leisure activities. Research suggests that parents are the primary reason a youth may choose to participate in an activity. However, if a youth decides to participate, it is facilitators, leaders, and peers who are more likely to influence persistence in the activity (Hultsman, 1993). In peer groups of adolescents, race more than gender may influence group approval ratings of various leisure choices (Philipp, 1998).

In terms of supporting parents' abilities to be effective facilitators (and supporters) of their children's learning they benefit from information about what counts as effective learning activities (in this case around math) and discourse that will scaffold children's learning and their own (Civil et al., 2007; Shumow, 1998). This means that parents also

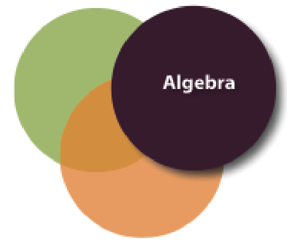


need mathematics education for themselves since it is unlikely that they have experienced positive discourse experiences and scaffolding around mathematics. There is also an opportunity for institutions like museums and community-based organizations to meaningfully engage parents and guardians in their children's learning, especially those in economically disadvantaged and underserved communities, where parents often do not get support nor are considered valued partners or positive resources in their children's learning (Civil et al., 2007; Csikszentmihalyi & Schneider, 2001). Again, outreach programs, community-based centers, and organizations can serve as intermediaries in this process (Hayes & Chodkiewicz, 2006).

As adolescents age and enter the work force, participate in school, homework, and out-of-school activities, and increasingly assume adult responsibilities they are feeling more and more time pressure. This pressure is exacerbated by societal expectations, as well as from increasing schoolwork. This time pressure is a growing concern since it limits an adolescent's ability to pursue free-choice learning/leisure activities; this is especially the case for girls (Hilbrecht et al., 2008).

Although not specifically identified in the scope of this review, gender issues, such as how gender affects learning styles in mathematics, may be worth further consideration. The literature in this area is extensive in school mathematics and includes researchers such as Elizabeth Fennema, Patricia Campbell, Judith Kleinfeld, and Joann Rossi Becker. Becker's "connected teaching" (see p.169 Table 20-3 of Becker, 1995) may provide some guidelines for facilitators working with girls on mathematics concepts. In general, recent findings support more active cognitive learning environments for boys and more group collaboration work for girls. Fennema and Peterson propose the Autonomous Learning Behavior model as a possible explanation for the development of gender differences in mathematics. This model suggests that observed gender differences are a result of females' lowered participation in autonomous learning behaviors which both require and develop one's ability to work independently in high-cognitive-level activities (refer to the references section for additional gender and mathematics references).

Algebra
Algebraic Thinking and Doing



In increasing numbers, policy makers have been calling for *algebra for all students*. Many states and school districts mandate it and universities require it for admission. In addition, for several years now, the National Council of Teachers of Mathematics, the Mathematical Sciences Education Board, and many other organizations have been exploring the nature of and role of algebra in the formal K−14 curriculum. As these groups, teachers, and others have explored this issue, they have struggled with the *meaning of algebra* and the *meaning of all is.* In a parallel fashion, there have been increasing efforts on the part of free-choice learning organizations to promote mathematics learning, including museums and community-based organizations such as Boys and Girls Clubs and Girls, Inc.

At its core, algebra is about identifying patterns and expressing them in some fashion. To engage in algebra, students must learn to collect data, organize those data, recognize patterns in the data, and express the patterns they've observed. Algebra is essentially a way of thinking, a way of interpreting, and a way of understanding situations. The world is full of regularities, relationships, and patterns that students can identify. *Algebraic thinking* is a vehicle for exploring the world, its regularities, relationships, and patterns. Children of all ages (elementary, middle school, and high school) can use *algebraic thinking* to investigate phenomena and relationships, particularly when this thinking involves engaging deeply and *doing algebraically* as well. The representation of these relationships may change according to the maturity and sophistication of the learner, but *algebraic thinking* is for all learners.

While there is a body of research related to *algebraic thinking* in the formal school setting, a search of the published research revealed no literature specifically focused on *algebraic thinking* in free-choice learning settings such as museums. Research, perhaps as a part of this project, could begin to address this issue. According to Kliman & Mokros (2001), future "research should: uncover the mathematics that a broad range of parents do 'naturally' with their children so that educators and parents can build on effective practice; investigate impact over time of interventions on different populations; and examine factors that contribute to parental support of reform mathematics." (Kliman & Mokros, 2001).

Although there was no specific research focused on algebra in museums, we did identify two evaluation studies that had been conducted on mathematics exhibitions, one on modern mathematics and the other about calculus. The evaluation of the modern mathematics exhibition *Beyond Numbers* (Korn & Ades, 1995) compared visitors' experiences with and conceptions of mathematics before entering the exhibition *and then after visiting* and also assessed visitors' perspectives about the overall experience, including what they thought the "big ideas" were. One-third of visitors could articulate a big idea with the most common theme being that math is part of everyday life. A few visitors talked about math as being more than numbers, more than what they learned in school, and found in art and nature. No visitors talked about math in terms of creativity and experimentation.

The evaluation of *Handling Calculus*, a set of interactive exhibits designed to engage visitors with kinesthetic, application-oriented, and concept-focused approaches to calculus and pre-calculus concepts, demonstrated that visitors associated the exhibits with school math, recognizing mathematics terms and symbols, reconstructing graphing skills, and recalling fragmentary concepts they had learned in school. Visitors recognized and valued the hands-on and whole-body approaches used in *Handling Calculus*, contrasting the exhibits with their paper-and-pencil school experiences. Some visitors wondered if they would have done better in school calculus if their teachers had used the approaches taken within the exhibition.

Both exhibitions evoked powerful emotional responses. In the Korn and Ades (1995) study, slightly less than one-quarter indicated that they had a confusing or boring experience, a significant number of people to report negatively given that generally there is a *courtesy bias* in this type of study, in which people report more positively in order not to offend. This suggests that the proportion of people who left feeling confused could be considerably higher. Researchers concluded that some of these visitors were likely "math phobic" and recommended having facilitators on the floor at all times, dedicated to helping visitors and answering questions. However, this study also showed positive emotional responses. For instance, even though few visitors discussed key messages, those who mentioned at least one idea did so with great enthusiasm and a sense of revelation, suggesting that ideas in the exhibition seemed new; a few even indicated that the exhibition changed the way they would look at the world. As previously suggested, findings from the *Handling Calculus* exhibit evaluation (Gyllenhaal, 2006) showed how visitors' experiences stimulated memories they associated with school mathematics, including recounting stories about former mathematics teachers and the ways in which they had been taught mathematics, as well as a range of both positive and negative emotions they associated with math.

There is also free-choice learning research in anthropology and mathematics education focused on everyday cognition which investigates how people use mathematics concepts naturally in their lives (see Saxe, 1988; Guberman, 1996; and Nasir, 2000). These include studies of street children in Mexico dealing with money handling and grocery shoppers making decisions around the cheese counter. Although this research shows that people often have rich, context-dependent knowledge of mathematics, like much of the learning in schools, this knowledge does not seem to transfer well to other situations, reinforcing the context-specific nature of learning. This has caused some school-based researchers to debate the usefulness of everyday mathematics in mathematics teaching (Carraher & Schliemann, 2001), although perhaps the question could be reversed also to ask how well school mathematics supports the use of math in everyday life.

Other free-choice learning research has focused on family and informal mathematics such as gaming. Edwards and Ruthven (2003) results show that children seem more capable than previously thought in identifying mathematics in many everyday activities (e.g., dressmaking, knitting, chess, pool, Lego), although they may not recognize it if the activity is considered too easy, there is no obvious problem to solve, or the solution is intuitive rather than formal. The mathematics identified tended to be fairly simple and in the form of shapes, angles, and measurements. Often children would recognize a type of mathematics used in one activity but not recognize that the same mathematics was present in another activity, even after just having identified it. Children also tended to see mathematics having only “one right answer” and missed seeing the mathematics when several answers or solutions were possible. For example, children saw dressmaking as making a dress to fit a particular size person and thus expected that the measurement for that dress size would have only one right answer, missing the fact that there were probably several different ways to lay out the fabric and fit the shapes together on one piece of fabric to optimally use the material for minimum waste (Edwards & Ruthven, 2003).

Two other studies investigated children playing an educational game called *Treasure Hunt*. Observations revealed that the children used collaboration and distribution of goals to solve the arithmetic problem at hand (Saxe & Guberman, 1998). More generally, children worked collectively to pursue distinctly different goals for the same activity (Guberman & Saxe, 2000). Based on their prior mathematical understanding (and we assume comfort level), they took different roles and used different materials resulting in different paths to solve the same overall problem. For example, one child chose to be a customer overpaying for an item, while the other child was the storekeeper calculating the amount owed and the change due. Thus different actions were possible in the same activity. Researchers concluded that although all participants were engaged in the same activity, the cognitive work accomplished collectively was distributed among group members (Guberman & Saxe, 2000).

These studies demonstrate that children may have mathematical ideas or goals in one situation but not in another seemingly similar situation. Children may not see the mathematics or be able to participate in the same way, even though adults understand that the activities share many similarities. These studies also highlight the need for facilitators to be aware of multiple zones of proximal development at work in the same space. This means helping parents, older peers, and facilitators understand the subtle balance between supporting and challenging children. The place at which this balance is achieved is the *zone of proximal development* (ZPD), the place where mastery and self-efficacy are more likely to occur (Rathunde, 2001). Creating the appropriate mix of optimal challenge and success, in other words finding the learner's ZPD, can be difficult. It may be helpful to test rough ideas with target audiences to co-construct activities.

There is considerable empirical research that has been conducted in school classrooms using a "captive audience" to examine *algebraic thinking and doing*, observing how students build knowledge, and how to best direct their thinking towards deeper understanding. Because research findings are context dependent and this research most often has focused on groups of students working on problems over a considerable length of time (at least compared to a museum experience) with in-depth facilitation, this research may not be directly transferable to free-choice settings, but it does point to important issues to consider. For example, in free-choice settings, the exhibits or activity areas need to be designed in ways that help visitors quickly identify the crux of the activity so that they can build upon their spontaneous *algebraic thinking and doing*. In addition, because of the nature of these settings, learners in free-choice contexts, particularly those in unfacilitated spaces, engage with the activities at hand from "multiple entry points" (e.g., *Beyond Numbers*) and thus the designer of the activity needs to understand and provide scaffolding for a variety of problem-solving approaches.

Environment

Research Focused on Free-Choice Learning Environments and Activities

The dominant motivation for humans is meaning-making and the need to make meaning of one's physical environment is powerful and innate. One adaptation driving meaning-making is the fact that humans are inordinately curious organisms. This drive is directly tied to learning since curiosity and learning represent a feedback loop; curiosity evolved in order to facilitate learning; learning occurs in order to satisfy curiosity (Kaplan & Kaplan, 1982). Curiosity is also driven by the need to explore and make sense of one's environment, and in turn, the desire to explore one's environment fosters and satisfies curiosity. Environments with "mystery," that foster moderate degrees of curiosity without being scary or overwhelming, that are complex and invite exploration, are far more desirable and appealing than are those without such qualities.

Directly related to curiosity is novelty, the construct used to describe unfamiliar environments, events, or objects. Beginning in the 1970s, Falk and his colleagues began studying the impact of environmental novelty on learning in out-of-school settings (Falk, Martin, & Balling, 1978; Martin, Falk, & Balling, 1981; Falk & Balling, 1982). They discovered that novel settings dramatically influenced learning. When settings were extremely novel or extremely familiar (e.g., boring), learning decreased. However, if an optimum amount of novelty was introduced, learning was enhanced.

Based on these findings, researchers have suggested that the learning that occurs in free-choice settings is often a direct result of novelty-seeking behavior as visitors orient themselves in space, explore environments that are unusual, and try to make overall sense of the environment. Moderately novel settings are stimulating and exciting thus fun. In fact, researchers have argued that because of their moderate yet "safe" levels of novelty, museums represent ideal places for learning that visitors intuitively seek out because of these very traits (Falk & Dierking, 2000). These basic findings have been replicated by a range of investigators in a wide variety of other free-choice learning settings (Kubota & Olstad, 1991; Anderson & Lucas, 1997).

One critical aspect of free-choice learning environments, be they physical or virtual, is the effort to optimize moderate novelty through the design process. Good design is appealing, draws learners in, engages all their senses, and compels them to investigate the topic at hand. Research suggests that free-choice learning also is optimized when there are opportunities for cultural, social, and cross-cultural interaction, thus spaces that are comfortable and inviting, and allow people to congregate, engage, and learn together, are effective (Falk & Dierking, 2000).

A prerequisite to cultural/social/cross-cultural interaction between children is the existence of spaces in which they can congregate, get involved, focus their undivided attention, and learn together. Spaces they can call their own like special rooms, gardens, or athletic courts work well (Loukaitou-Sideris, 2003). Larger spaces that allow for games, group projects, and even cultural festivals are also important for cultural/social/cross-cultural interaction (Loukaitou-Sideris, 2003).

In addition to the design of spaces, the design of the activities in which people will engage in these spaces is important, particularly since good design is increasingly moving away from the concept of exhibitions as visual storage of objects and “books on a wall,” to the view of exhibitions as environments and spaces in which visitors can actively engage and participate in the activities at hand. Activities in free-choice learning settings start with the personal interests of visitors, afford collaboration and talk, and often include elements of fantasy and imagination. Activities that are purposeful, and offer choice and ownership are also important. Embedding activities into relevant everyday life, using narrative and emotional elements can also foster interest and engage learners of all ages.

The characteristics that seem to best support knowledge construction in out-of-school settings often include: (a) a familiar context; (b) a problem or dilemma driving the activity; (c) purposeful and goal-directed activity; (d) the use of the learner’s own natural language; and (e) creation of apprenticeship-like opportunities affording observation of the skill and thinking involved and mentorship by a more expert person (Lester, 1989; Masingila et al., 1996).

Familiarity and immediate relevance are very important for initial engagement (Masingila et al., 1996), while fun and playfulness are important for continued engagement and persistence without stress. *Fun* is a relative term that can refer to opportunities for social interaction, physical movement, and/or the use of imagination. What is clearly not fun for adolescents is learning without relevance (Stinson, 1997). A focus on playfulness alleviates boredom, releases tension, and prevents aggression, while also promoting group membership and civic engagement (Staempfli & Mannell, n.d.). Dewey (1913) recommended that a goal-directed activity embedded within a fun and playful atmosphere may be best for effective learning (Rathunde, 2001). Some ideas for how to be immediately relevant and connect quickly with audiences include the use of popular culture as a context (for example, the television show *Numb3rs*), creating novel twists with familiar elements or objects, and so on. To hook and engage learners quickly is very important with all audiences. It is even more critical

with family visitors who vote with their feet. This is something that can be tested, both with Boys & Girl's Club (BGC) youth and family visitors to OMSI, by creating moveable carts or other mechanisms with rough activities or even just objects to see what captures attention and resonates with these groups. In this way one will be co-creating activities with youth and families.

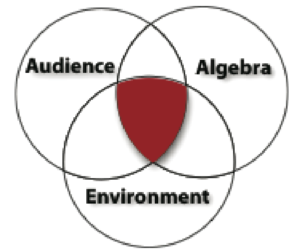
There is conflicting information about whether an approach of using specific problems versus open-ended activities versus multiple entry points is most effective. Research would suggest that the best strategy is to work to have a mix of types of activities (very open-ended activities, some more directed, some requiring facilitation, some not needing facilitation at all, etc.) with a variety of ways that visitor(s) can go about figuring out and engaging in algebraic thinking (music, art, collaboration, etc.).

Relevance can be fostered in many ways but two approaches that seem to work well with youth include music and the use of historical accounts and narratives. Music is an excellent vehicle that fosters meaningfulness and emotions in youth, particularly if they get to make and record the music themselves. In a study by Batt-Rawden and DeNora (2005), a CD that was meaningful to the researcher was used as a "sympathetic entry point" for discussion and facilitating narratives of the participants interests, opinions, and values. The authors referred to the process of musically creating social worlds, identities, and situations as *musicking* and found that people unfamiliar with one another found a common interest in the music and were able to build social capital together (Batt-Rawden & DeNora, 2005).

Cultural relevance can also be provided through historical accounts. One approach is to situate activities within their historical context showing that historical contributions to mathematics can be found in African, Aboriginal, Hispanic, and Asian cultures. For example, Africans were the first to use numerals, the Chinese invented negative numbers, Native Americans first conceptualized and used the symbol for zero, and the word *algebra* is Arabic (Ezeife, 2003). Narrative is a good way to provide cultural context, create interest, and engage learners of all ages. Narratives appear to be developmentally significant for learning. They serve both social and cognitive functions, are highly motivating, focus on social relationships, and are socially dynamic among and between participants (Gauvain, 2005).

Such narratives can also be virtual. For example some researchers are creating interactive (online) narrative learning environments (NLE) to promote co-construction of the narrative, exploration of the learning task within the narrative, and reflection or analysis of the consequences. Examples of NLEs include virtual story telling, interactive drama, and participatory narratives (Sarmiento et al., 2008).

Online chat or other online synchronous or asynchronous discourse communities may be a way to use/encourage narrative, for example older visitors could be encouraged to reflect upon their algebra learning by telling stories. In this and other ways online environments can extend the exhibit experience while also fostering cultural relevance.



Recommendations

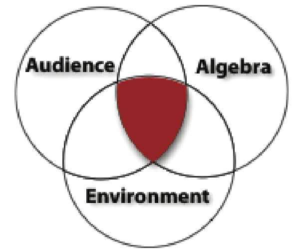
The key to an effective exhibition will be to maximize as much as possible and integration of audience needs, the richness of mathematical /algebraic thinking and doing, and use of effective learning strategies and design approaches. The following recommendations emerge from initial research synthesis and target this overlap.

- Create exhibition spaces that are comfortable and inviting, draw learners in, engage all of their senses, provide opportunities for cultural, social, and cross-cultural interaction.

- Create small spaces such as special rooms or gardens that youth can call their own; they are likely to gravitate to these spaces. One possibility is nooks inside areas in which 2--4 children or one family could converse or experience the exhibit together where more time can be spent in conversation without feeling pressured by other visitors. A great example is at the Vancouver, B.C., aquarium in the children's area where a small 'cave' is attached to the big fish tank allowing a few kids or two adults to escape the crowd, lie down in the carpeted space, and look up into the tank.

- Create larger spaces that allow for games, group projects, and even cultural festivals that can encourage cultural/social/cross-cultural interaction.

- Design exhibition spaces and train facilitators to foster a culture of figuring out (algebraic thinking), in which no one feels dumb and there is respect for diverse ideas, multiple solutions, multiple strategies, and multiple entry points; quality professional development should help facilitators learn how to ask good questions and encourage inquiry, rather than assuming the role of teacher. Testing can help the team investigate the optimal conditions, materials, and activities that can foster such a culture.



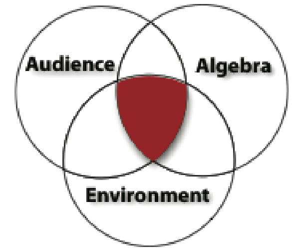
- Exhibition experiences focused on mathematics often evoke powerful emotional responses (both negative *and* positive), design exhibit spaces that acknowledge and ameliorate negative emotions and build on positive emotions. Possible ideas include creating spaces that are cozy, comfortable, and inviting with enough seating, use colors that influence positive emotions, as well as deal with emotions directly through the use of comment boards or humor (one thought was to create "Lucy's Booth," a place where you can share either your positive or negative emotions about math).

- Associations of mathematics with school experience can be both negative and positive; several researchers have shown a relationship between school science and mathematics learning and a decrease in interest for learning science and mathematics. The design team should be cautious about directly associating the exhibition with school learning.

- Provide many and varied opportunities for visitors to personally identify with the exhibition through images and the "natural language" of people in the exhibition who look like and talk like "cool" adolescents or young adults; (e.g., the *Numb3rs* TV show).

- Research suggests a number of specific elements of effective exhibit activities:

    - Start with the personal interests of visitors
    - Are purposeful yet playful and fun, thus perceived as serious play
    - Empower learners through collaboration and talk, and may include elements of fantasy and imagination
    - Are embedded into relevant and familiar everyday life situations, using narrative and emotional elements to pull learners in to foster interest and engage learners of all ages
    - Explicitly point to the mathematics (algebraic thinking) in everyday life and how it relates to the prior understandings and experience visitors bring to the exhibition
    - Include a problem or dilemma at the centerpiece of the activity
    - Offer choice and control over the what, where, how, and with whom of learning by providing multiple entry points, multiple solutions, and multiple strategies which allow individuals to decide for themselves how to best use the available resources for their own learning



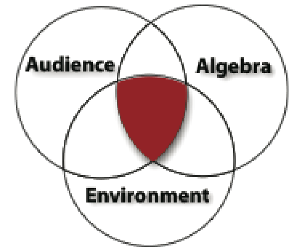
- Offer optimal challenge and are open-ended enough for multiple approaches and solutions (not just "one right answer"), focus on exploring the concepts and ideas not on getting the "right answer" to a <u>single problem</u>
- Should not be considered too easy and yet reinforce the notion that solutions in mathematics can be intuitive and/or formal
- Have explicit goals for the visitor (e.g., to look for patterns, relationships, and rates of change)
- Provide playful, positive but constructive feedback on progress towards goals and concepts
- Provide apprenticeship-like opportunities and mentorship by a more expert person; however, the focus should be on asking appropriate questions not being "the teacher"
- Encourage and foster peer to peer collaboration in which children take turns and build on the content of their conversations; peer groups of younger and older children all benefit physically, intellectually, and socially; one possibility is to involve Boys and Girls Club youth as facilitators in the exhibition or exhibition developers which could help reduce the intimidation level of the exhibition

- Games, puzzles, stories, and group projects that result in real products are potentially important avenues since they afford opportunities for active engagement in algebraic thinking through collaboration, conversation, use of imagination and fantasy and narrative approaches and, if designed well, include many of the elements above.

- There may be ways to have activities incorporate feedback from previous visitor groups, extend stories, challenges over time, or continue to interact remotely using technology.

- The development team should explore innovative ways of using technology to extend the experience beyond the walls of the museum and as advance organizers accessible prior to the exhibition experience (e.g., IPod downloading, computer chats/online discourse communities, MySpace).

- Some research specifically refers to musically creating social worlds, identities, and situations, a process called *musicking*; visitors could explore algebraic concepts by creating music and sharing their music with other visitors; this could include a community building aspect in which visitors that may not usually encounter each other could find a common interest in music.



- Important factors to include in the professional development component of this project include:

    - Adolescents learn best when supported and challenged individually at their own level of understanding, so facilitators need to be aware of multiple zones of proximal development at work in the same space at the same activity
    - It can be challenging for facilitators who are not parents or friends to easily recognize each individual *zone of proximal development*; activities need to be designed with multiple entry points which allow learners themselves to select, to gravitate towards their own optimal challenge and to decide for themselves how to best use the available resources for their own learning
    - Make facilitators aware of perceived group status in terms of comfort with and understanding of algebraic thinking since such status can affect learning in groups even at early ages; they should be trained to act as facilitators not "teachers"; their role is to ask questions not to provide the "right answer"
    - Be warm, caring, and attentive in order to use the adolescents' interests and relevant life experiences to draw them into deeper thinking around the content
    - Design quality professional development experiences which help facilitators learn how to ask good questions and encourage inquiry, rather than assuming the role of teacher; testing can help the team investigate the optimal conditions, materials, and activities that can foster such a culture

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Annotated Bibliography

(Note: The * denotes "must reads." Some annotated references may not be in the literature discussion but were included in case they might be useful.)

Anderson, A. (1997). Families and mathematics: A study of parent-child interactions. *Journal for Research in Mathematics Education*, 28(4), 484--511. From abstract: "Explores mathematics and parent-child interactions in a group of 21 parents and their four-year-old children. Findings indicate a wide range of mathematics displayed with counting being the most prevalent activity. All parents succeeded in injecting some mathematics in most sessions. Questioning children's knowledge was found to be the main technique through which parents elicited mathematics."

Batt-Rawden, K., and DeNora, T. (2005). Music and informal learning in everyday life. *Music Education Research*, 7(3), 289--304. An interesting aspect of this study was the use of a CD of songs presented to the participants by the researcher. The CD was meaningful to the researcher and was used as a "sympathetic entry point" for discussion and facilitating narrative accounting of the participants' interests, opinions, and values. Six subsequent CDs used in the study were designed by the participants. The authors call the process of musically creating social worlds, identities, and situations *musicking*. The *participatory design* process actively involves the end users in the design process to help ensure that the product designed meets their needs.

Carraher, D. W., and Schliemann, A. D. (2001). Is everyday mathematics truly relevant to mathematics education? In J. Moshkovich & M. Brenner (Eds.), *Everyday and academic mathematics in the classroom: Monographs of the journal for research in mathematics education, 11*. Reston, VA: The National Council of Teachers of Mathematics. This monograph discusses the issues around using everyday math research to inform classroom practice and its usefulness to higher more abstract levels of mathematics learning. The authors feel that ultimately math has to provide tools for going further or understanding more abstractly than what one would learn in an everyday situation. "The outstanding virtue of out of school situations lies not in realism but rather in meaningfulness. Mathematics can and must engage students in situations that are both realistic and unrealistic from the student's point of view.

But meaningfulness would seem to merit a consistently high position on the pedagogical pedestal."

Ciardiello, A. V. (2006). *Puzzle them first!: Motivating adolescent readers with question-finding*, 100. International Reading Association. This book is more school based, but could be a resource for exhibit developers and facilitators.

Civil, M., Guevara, C., and Allexsaht-Snider, M. (2002). Mathematics for parents: Facilitating parents' and children's understanding in mathematics. *24th Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. A look at the impact of intervention programs for parents supporting their children's learning in a reform math classroom. Parents are viewed as intellectual resources and not with a deficit model. "Parents also need opportunities for meaningful learning of challenging mathematics content," and "our findings suggest that giving parents opportunities to actively construct their own understanding of mathematics concepts provides a critical foundation for their work with their own children." The authors recommend a critical examination of bilingual teaching and learning environments. They are learning the language of mathematics along with English as a second language.

Csikszentmihalyi, M., and Schneider, B. (2001). Conditions for optimal development in adolescence: An experiential approach. *Applied Developmental Science*, 5(3), 122. This is the introduction to this journal issue. The overall message is that parents continue to influence the development of their children into the teenage years. In reference to the articles "Perhaps their strength lies in their identification of parenting practices that promote adolescent well-being. Too often, parents assume that their parenting role ceases when their children learn to drive." The following are some points made in the journal: The teenage years are much more complex than "turbulent." While an internal locus of control and self esteem steadily increase with age, positive affective states decrease. Self worth decreases until 15 or 16 then increases. Children who see their families as providing both challenge via expectation and opportunities for excellence and support (warmth and caring) spend more of their day in conditions of undivided attention. Growth-conducive experiences are ones that include intrinsic motivation, concentration, goal-directness, and productive activity.

Denner, J., and Werner, L. (2007). Computer programming in middle school: How pairs respond to challenges. *Journal of Educational Computing Research*, 37(2), 131--150. This article describes how middle-school pairs of girls in an after-school and summer program thought about the problems they had while programming and the strategies they used to solve them. “In successful collaborations, conversational turns build on each other and the content moves the pair closer to solving the problem.”

***Edwards, A., and Ruthven, K. (2003). Young people’s perceptions of the mathematics involved in everyday activities. *Educational Research*, 45(3), 249--260.** Previous researchers found participants did not record an activity as mathematical if there was no obvious problem to solve. And “as soon as an activity became easy it is seen as ‘common sense’ rather than mathematics.” In this study, year seven (UK) pupils liked “maths” more than did year 10 pupils. “The results of this study suggest that young people may be capable of identifying the mathematics embedded in a wider range of everyday activities than previously thought, and that the factors influencing their mathematical representations are very complex.” The results did not support the view that young people are less likely to identify mathematics in activities they associate with females (dress-making, knitting). Also there was no evidence to support a link between associating an activity with intelligent or challenge-seeking people and identifying mathematics within that activity.

***Ezeife, A. N. (2003). Using the environment in mathematics and science teaching: An African and aboriginal perspective. *International Review of Education*, 49(3/4), 319.** This article discusses the need for integration of lived experience into school math and science. Factors such as historical development of mathematics concepts, as well as folklore, myth, and legends, can root mathematics in culture other than European and European American. A book by Hatfield et al. (1997) documents the historical contributions to mathematics that can be found in African, Aboriginal, Hispanic, and Asian culture. For example Africans were the first to use numerals, the Chinese invented negative numbers, Native Americans first used the symbol for zero, and the word *algebra* is Arabic. See Hatfield, M., Edwards, N. and Bitter, G. (1997). *Mathematics Methods for Elementary and Middle School Teachers.* Boston: Allyn and Bacon. Many more examples of incorporating other cultures into math and science are discussed and referenced.

***Gauvain, M. (2005). Sociocultural Contexts of Learning. In A. Maynard and M. Martini (Eds), *Learning in cultural context: Family, peers, and school.* New York: Kluwer Academic Publishers:** This article reviews and synthesizes research on sociocultural influences on cognitive development. One relevant key finding in recent years is the important role older siblings play in children’s learning. Another is that Piaget’s idea that conflict (social or cognitive dissonance) leads to learning may be true for some cultures but perhaps not in others that value cooperation more highly. Some research implies that, for older children, collaboration benefited children's learning given a tendency in the learner to be systematic and strategic. "This pattern suggests that opportunities to learn from collaborative interactions are not independent of learning-related behaviors that children bring with them to social problem solving situations." Narratives appear to be developmentally significant for learning especially for younger children. Narratives have both social and psychological functions in learning, are highly motivating, include social relationships, and are socially dynamic between participants. One large difference in adult verses peer instruction is verbal exchange. While the adult may have more language to help the learner, the peer may provide a smaller status differential enabling the learner to steer instruction to better meet their needs.

Gray, P., and Feldman, J. (2004). Playing in the zone of proximal development: Qualities of self-directed age mixing between adolescents and young children at a democratic school. *American Journal of Education*, 110(2), 108--146. This study documented naturally occurring interactions between adolescents and younger children. Common interests and play styles, personal attraction, and complimentary desires to nurture and be nurtured drew the children together. Findings show both the older and the younger child benefited by the interaction physically, intellectually, and socially. Adolescents provided for a zone of proximal development in which the younger could benefit while the younger provided stimulus for the older to make their thinking explicit, be creative, and practice leadership.

Guberman, S. R. (2004). A comparative study of children's out-of-school activities and arithmetical achievements. *Journal for Research in Mathematics Education*, 35(2), 117--150. This study takes a comparative look at Korean American and Latin American families and their out of school engagement in activities with money and arithmetic and the relationship of these activities to arithmetic achievement. Both groups of parents shared high expectations and an emphasis on the importance of effort in achievement. The Korean American parents were more dissatisfied with their children's performance expressing their desire for their children to outperform their classmates.

Guberman, S. R., and Saxe, G. B. (2000). Mathematical problems and goals in children's play of an educational game. *Mind, Culture and Activity*, 7(3), 201--216. Using a game called treasure hunt the researchers looked at children's play and its role in learning. "...Children created and recreated thematic divisions of labor as they took on roles as storekeeper and customer. Thematic divisions were of particular interest because they enabled the accomplishment of mathematical problems that were beyond the independent ability of many individual players." And although problem solutions were accomplished jointly children became engaged in routines that created distinct goals for the same activity (division of labor).

***Gyllenhaal, E. (2006). Memories of math: Visitors' experiences in an exhibition about calculus. *Curator*, 49(3), 345--364.** **Abstract:** *Handling Calculus* is a set of interactive exhibits about mathematics developed by the Science Museum of Minnesota and TERC. The exhibits are designed to engage visitors with kinesthetic, application-oriented, and concept-focused approaches to calculus and pre-calculus concepts. Visitor interviews conducted during the evaluation of the exhibits, revealed how often visitors' experiences with the exhibits stimulated memories they associated with school math. They recognized math terms and symbols, reconstructed graphing skills, and recalled fragmentary concepts they had learned in school. In addition, they recounted stories about their former math teachers and the ways in which they had been taught mathematics, as well as a range of both positive and negative emotions they associated with school math. Visitors recognized and valued the hands-on and whole-body approaches used in *Handling Calculus*, contrasting the exhibits with their paper-and-pencil school experiences. Some visitors wondered if they would have done better at school calculus if their teachers had used the approaches taken by *Handling Calculus*.

Hadjioannou, X. (2007). Bringing the background to the foreground: What do classroom environments that support authentic discussions look like? *American Educational Research Journal*, 44(2), 370--399. A qualitative case study of a 5th grade classroom with authentic "Book Talk" discussions. Findings describe seven aspects of a classroom conducive to authentic discussion: physical environment, curricular demands, enacted curriculum, teacher beliefs, student beliefs about discussions, relationships among members, classroom procedures, and norms of classroom participation. "Classroom communities that aspire to foster authentic discussions must find their own rituals, norms, and balances." However this classroom provides one good example.

Hayes, D. and Chodkiewicz, A. (2006). School-community links: Supporting learning in the middle years. *Research Papers in Education*, 21(1), 3--18. This research shifts the focus away from schools and towards communities by including the perspectives of parents, community workers, and students at risk of disengaging from schooling. "Most of the parents indicated that they wanted to get help to support their child's learning as they did not feel that they had the skills to address these issues, and they felt powerless to stop their move towards disengagement." The parents viewed the teachers as the means to help their children. The study showed these parents had high levels of problem solving ability and perseverance and should not be viewed as uninterested or unable to help their children as is commonly the view. Parents view the main barriers to helping their children to be a lack of communication with teachers, a lack of information from schools about educational programs, language barriers, and a lack of interest by school staff. "A core consideration in publicizing and promoting sites of learning beyond schools is the need to challenge deficit views of families and communities, especially those views held by school-based personnel. A particular concern is that teachers and school executives do not generally perceive parents or their local communities as resources for learning."

Hektner, J. M. (2001). Family, school, and community predictors of adolescent growth-conducive experiences: Global and specific approaches. *Applied Developmental Science*, 5(3), 172. This study tested growth-conducive experiences for adolescents using structural equation modeling. "Results imply that teachers, parents, and communities can promote positive youth development by maintaining an environment rich in interpersonal support, autonomy, and opportunities to pursue challenges related to future goals."

Hilbrecht, M., Zuzanek, J., and Mannell, R. (2008). Time use, time pressure and gendered behavior in early and late adolescence. *Sex Roles*, 58(5/6), 342--357. This study assessed whether there were gendered patterns in the way teens spent their time. "Throughout the week girls experienced higher levels of time pressure than boys. The significant gender gap in time pressure is apparent during early adolescence and intensifies with age."

Hufferd-Ackles, K., Fuson, K. C., and Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81--116. This article provides description of one case where a math teacher changed her classroom from a traditional approach to a math-talk community. The authors provide a framework for helping other teachers make the same change. "Like previous research, we have argued that opening up one's classroom to student's ideas is the critical first step in achieving a discourse community."

Hultsman, W. Z. (1993). The influence of others as barrier to recreation participation among early adolescents. *Journal of*

***Leisure Research*, 25(2), 150.** This study examines perceived constraints to leisure participation identified by adolescents. Parents, especially mothers, remain a large influence on an adolescent's leisure "formal" or "institutional" recreation. They are still the ones that screen and qualify program opportunities. "Parents were perceived as the strongest human influence in decisions not to join an activity in which one has an interest." However parental influence was perceived to decrease with age. The decision to stop an activity that one has already participated in was mostly due to perceptions about the group leader or non-parental adults participating in the activity.

Kieren, C. (2007). *What do we know about the teaching and learning of algebra in the elementary grades?* Retrieved from http://www.nctm.org/news/content.aspx?id=12326**.** Relevant to informal math learning—"encourage young students to make algebraic generalizations without necessarily using algebraic notations." There is no description of how to do this. However there is a reference to a powerpoint presentation titled *The potential of geometric sequences to foster young students' ability to generalize in mathematics* by Moss et al. (2005). It does not look complete but can be found at: http://www.brookings.edu/gs/brown/algebraicreasoning/Moss_Presentation.pdf

Kieren, C. (1992). The learning and teaching of school algebra. In D. Grouse (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: MacMillan Publishing Co. This chapter starts with a historical overview which helps to understand why we humans have a hard time moving from the more rhetorical and syncopated algebra (a letter stands for the unknown quantity) to the symbolic form (letters stand for both given and unknown quantities). This took roughly 1300 years to happen the first time! Also included are some psychological considerations and cognitive research to date in 1992.

Kliman, M. (2006). Math out of school: Families' math game playing at home. S*chool Community Journal*, 16(2), 69. In this study families were given a geography game that incorporated elementary school math concepts (state statistics). Families were contacted twice about their experiences playing the game. Relevant results included: parents did not associate playing the game with school learning but with things that were intrinsically worth learning, parent participation declined with children's age, and parent education and occupation did not relate to extent of play. The authors identified two future research questions: 1) what factors contribute to family engagement in math-related activities at home? and 2) in what ways can various sorts of informal math activities with parents at home impact children's attitude toward and performance in related school subjects?

Kliman, M., and Mokros, J. (2001). ***Parents as informal mathematics teachers of their elementary grades children*****. Retrieved from http://metromath.org/library/uploads/ParentMathConference.pdf.** A report identifying future research needs. "While many parents know what it means to read with children, they're less certain of what it means to do mathematics." The conference concluded future "research should: uncover the mathematics that a broad range of parents do 'naturally' with their children so that educators and parents can build on effective practice; investigate impact over time of interventions on different populations; and examine factors that contribute to parental support of reform mathematics."

Korn, R. and Ades, S. (1995).** ***Beyond Numbers: **A Summative Evaluation. Unpublished report for Maryland Science Center. Alexandria, VA : Randi Korn and Associates.** This report presents findings from a summative evaluation of *Beyond Numbers*, an exhibition about modern mathematics. Funded by the NSF and IBM it was developed collaboratively by MSC and mathematics faculty at The George Washington University and its goals were for visitors to understand that: (1) there is more to modern math than they learned in school; (2) math is more than numbers; (3) math involves discovery, creativity, experimentation, play, and solving problems; and (4) math can be found in the everyday world, including in art, architecture, and nature. The evaluation compared visitors' experiences with and conceptions of mathematics before entering *Beyond Numbers* and then after and also assessed visitors' perspectives about the overall experience, including what they thought the "big ideas" of the exhibition were. Data was collected using a card sort to compare visitors' attitudes toward math before and after being in the exhibition, and open-ended interviews. "Math can be fun" was the only statement in the card sort in which a statistically significant difference was observed. However, there was an increase in the number of people who agreed with four additional statements ("Math relates to almost everything I do," "Math can be fun," "I like math," and "I can think like a mathematician"). Some of the attitude statements did not change including: ("Math never changes," "I want to learn about some mathematical ideas," "Math bores me," "I hope never to see math again," "People like me can never do math," "Math helps you understand nature," and "I want to learn about some mathematical ideas"). One-third of visitors could articulate one of the four main messages with the most common theme being that math is part of everyday life. A few visitors talked about math as being more

than numbers, more than what they learned in school, and found in art and nature. No visitors talked about math in terms of creativity and experimentation, a finding corroborated by the card sort findings. Although few visitors discussed key messages, those who mentioned at least one idea did so with great enthusiasm and a sense of revelation, suggesting that ideas in the exhibition seemed new; a few even indicated that the exhibition changed the way they would look at the world. Over half had a fun or educational experience, indicating enjoying being challenged and having the opportunity to use their minds in a museum exhibition. Slightly less than one-quarter indicated that they had a confusing or boring experience, a significant number of people to report negatively, given that generally there is a "courtesy bias," in which people report more positively in order not to offend, suggesting that the proportion of people who left feeling confused could be considerably higher. Researchers concluded that some of these visitors were likely "math phobic" and recommended having facilitators on the floor at all times, dedicated to helping visitors and answering questions.

***Loukaitou-Sideris, A. (2003). Children's common grounds. *Journal of the American Planning Association*, 69(2), 130.** This is a study that examines cases of successful common grounds. These are settings used harmoniously by children of different racial and ethnic backgrounds. The cases include a school, a community center, and two parks. Previous research shows "children are profoundly shaped by the sociophysical conditions that surround them...children's reactions to the spatial dimensions of their environment are often more visual, emotional, and spontaneous than those of adults." Second to time spent together, the most important prerequisite for successful interactions was the existence of special spaces that children could call their own. In these spaces they could congregate and become involved in educational, recreational, and social activities.

Luna, B. (2004). Algebra and the adolescent brain. *TRENDS in Cognitive Psychology, 8(10),* 437-439. This article stretches implications for algebraic thinking a bit but it is safe to say that brain areas identified as relating to executive function and social cognition undergo major reconstruction during adolescence. Plasticity of the brain at this time may relate to different ways of thinking to solve the same problems as adults. You could not say one way is better than the other. However keeping in mind that not only do children think differently than adults (as Piaget's work illuminates) but also adolescents developing more abstract thinking may think differently still.

Masingila, J. O. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, 31(1-2), 175-200. This was largely an article about examples of differences between in school math learning and everyday math learning and a framework for informing classroom practice. This synthesis seemed relevant: "Knowledge constructed in out-of-school situations often develops out of activities which: (a) occur in a familiar setting, (b) are dilemma driven, (c) are goal directed, (d) use the learner's own natural language, and (e) often occur in an apprenticeship situation allowing for observation of the skill and thinking involved in expert performance (Lester, 1989)."

Matthews, M. W., and Kesner, J. (2003). Children learning with peers: The confluence of peer status and literacy competence within small-group literacy events. *Reading Research Quarterly*, 38(2), 208. Individuals within social groups in classrooms experience interactions around content with their peers in different ways. Some find the interactions supportive and some find them stressful. Either way a child's perceived status in the group influences his or her learning. "If a child experiences success or failure during these interactions ...this affects substantively a child's perceptions of self as well as how a child interprets future events… ."

Moriarty, V. (2001). Family learning programmes: An investigation of parental perceptions, social capital and social inclusion. *Educate*, 1(1). This is an analysis of parental perceptions of the efficacy of family learning programs in the UK, however, most of the data is evaluative in nature and thus may not be generalizable. They aim to encourage parental involvement with their children's education and the development of parental basic skills. All parents in this study felt they had a pedagogical role in their children's lives regardless of economic class or education. The parents felt they benefited by the program, had an increased sense of efficacy in their parenting, and felt more confident about being in the classroom and talking with teachers. However, the program's goal of social inclusion could not be assessed.

***Nasir, N. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning*, 4(2/3), 213--247.** The author argues that the formation of goals and identities in practice are related processes that are central to learning. "Both the shift (with age) in goals and the shift in identities were related to the emergence of new ways in participating in practice. This shift in identity was rooted in the changing nature of engagement." African American students showed increasing participation and learning of mathematical knowledge with age in activities they identified with. This is the opposite for school mathematics learning. African American students tend to “disidentify” with school mathematics. “The patterns of decreasing identity may be comprised of engagement, alignment, and imagination processes on a local level and related to the mathematical goals students do and do not construct in the classroom: goals that are fundamentally linked to students’ learning of mathematics.”

Nesdale, D., and Flesser, D. (2001). Social identity and the development of children’s group attitudes. *Child Development*, 72(2), 506. “Consistent with SIT (social identity theory) and research with adults, the results indicated that children as young as five years of age were sensitive to the status of their social group, and that ingroup status has important implications for both their desire to remain group members as well as their perceived similarity to other groups members.”

Philipp, S. (1998) Race and gender differences in adolescent peer group approval of leisure activities. *Journal of Leisure Research*, (30)*2*, 214-232. A quantitative study on the relationship of gender and race on adolescent peer group approval of leisure activity. Since the study was in the southern US and only on older adolescents, it is probably not very generalizable to the Pacific Northwest 10-14-year-olds. Also household income was not measured and could also be a factor. However these findings indicate race more than gender influences peer group approval ratings for a given leisure activity. Specifically for “going to museums” there was a weak approval rating for both African American and European American adolescent peer groups. “For example, both racial groups were much more likely to find going to the beach, mall, or watching television to be accepted activities, and going to a museum, golfing, or collecting stamps and coins to be non-accepted activities.”

Rathunde, K. (2001). Family context and the development of undivided interests: A longitudinal study of family support and challenge and adolescents’ quality of experience. *Applied Developmental Science*, 5(3), 158. “Rogoff suggested that a parent’s central job is to guide children’s participation in activities in an optimally challenging way ‘through material arrangements of children’s participation in activities and responsibilities as well as through interpersonal communication, with children observing and participating at a comfortable but slightly challenging level’ (p. 18). Thus parents must learn when to support and when to challenge their children to keep them in the *zone of proximal development*, a zone of optimal arousal wherein the growth of mastery and self-efficacy are more likely to take place.” Optimal arousal is more likely to occur in a family that is both supportive and challenging. Supportive parents are warm, affectionate, attentive (listening), open and non-judgmental, and helpful. Challenging parents model self-directed behavior, set rules and limits, and expect the child to achieve goals, in other words focus their attention. Dewey refers to this as a balance between play and work. All play is foolishness. All work is drudgery. The optimal state for learning is to be playful and serious at the same time. Here this playful-seriousness is called *undivided attention*. The findings of this study show "adolescents most likely to spend time in undivided interest were those from high-support/high-challenge families." And "the mixed messages of increasing support and decreasing challenge, and decreasing support and increasing challenge, (over a 2-year period), led to the largest decline in adolescent's concentration on goals." This study supports previous research that shows permissive or authoritarian families lead to negative experiences while authoritative families (balance support and challenge) lead to positive experiences.

Sarmiento, J., Trausan-Matu, S., & Stahl, G. (2008). *Co-constructed narratives in online, collaborative mathematics problem-solving*. Retrieved from http://idea.library.drexel.edu/handle/1860/1727**.** Creating a Narrative Learning Environment (NLE) will promote co-construction of the narrative, exploration of the learning task within the narrative, and reflection or analysis of the consequences. NLE's include virtual story telling, interactive drama, and participatory narratives.

Saxe, G. B., and Guberman, S. R. (1998). Studying mathematics learning in collective activity, *Learning and Instruction*, 8(6), 489-501. "Our analysis of Treasure Hunt point to various factors that influence the formation of mathematical goals in collective activities. First, children's prior mathematical understanding appear to have influenced their adoption of thematic roles in play...Second, the artifacts used in Treasure Hunt—like gold doubloons—also influenced the emergence of thematic roles in play...Finally, the activity structure of play, in which social norms supported the distribution of problem solutions, also is implicated in goal differentiation." By taking roles in play based on their prior mathematical understanding (and we assume comfort level) and using the play to take different paths in problem solving the same problem, the children used collaboration and distribution of goals to solve an arithmetic problem.

Shaw, S. M., Kleiber, D. A. and Caldwell, L. (1995). Leisure and identity formation in male and female adolescents: A preliminary examination. *Journal of Leisure Research*, 27(3), 245. A quantitative study of the role of leisure activity in the identity formation process. Specifically the relationship between participation and level of identity development. Results indicate that females showed a positive association between participation in sports and physical activities and identity development. Males did not show this positive association for sports and physical activity. Furthermore males showed a negative association with time spent watching TV and identity development while females did not. Participation in social and other free time activities did not show a significant association for either gender.

Shumow, L. (1998). Promoting parental attunement to children's mathematical reasoning through parent education. *Journal of Applied Developmental Psychology*, 19(1), 109-127. This study investigated parental education about math reform. All parents in the study received newsletters with information about reform curriculum. Some parents also received personable phone call conversations with the math educators. "Initially all parents were highly directive and controlling of their children's solutions." All parents decreased in directive control and increased in collaborative guidance. However the "conversation-added" group decreased to a greater extent and displayed more knowledge about their children's mathematical reasoning. This shows that parents benefit in the same ways as children by positive social interaction around content and by learning through text combined with dialog rather than text alone.

Simpkins, S., Davis-Kean, P., and Eccles, J. (2005). Parents' socializing behavior and children's participation in math, science, and computer out-of-school activities. *Applied Developmental Science*, 9(1), 14-30. This quantitative study looked at two-parent European American middle-income families. They measured for parental behavior as predictors of child's participation in computer, math, and science activities. Unfortunately this study seemed to define math and science learning very narrowly (i.e., looked for "math related books," or "science activity related materials") possibly missing much of the everyday math and science learning activity. Fathers who participated in children's activity had a higher level of education and family income than fathers that did not participate. "In two-parent families, mothers and fathers' behaviors are related to how children use their time after school. In addition, mothers' and fathers' behaviors are not independent of each other; rather, parents' behaviors work in tandem." And "our results suggest that for mentally challenging activities, behaviors that include direct parent-child interaction are more strongly related to young children's engagement in these activities."

Staempfli, M., and Mannell, R. C. (n. d.) Adolescent playfulness and well-being. Retrieved from http://www.lin.ca/resource/html/cclr%2011/CCLR11-143.pdf**.**

"Playfulness has the capacity to alleviate boredom, release tension and prevent aggression, and to promote group membership and civic engagement, which may enhance positive experiences at school, at home or during leisure."

Stinson, S. (1997). A question of fun: Adolescent engagement in dance education. D*ance Research Journal,* 29(2), 49-69. An interpretive research study in which the author identifies "fun" as a main criteria for engagement for adolescents in this study. Fun can have different meaning to different adolescents and different still from adults. For adults it tends to mean "just for fun" and implies not of much value. However for these adolescents "fun" meant social interaction, "making up stuff" (use of imagination), moving around, and learning. This represents another way to look at the balance between playfulness and seriousness. In fact the author uses Csikszentmihalyi's idea of flow or optimal experience throughout her discussion. What was clearly not fun for these adolescents was when learning was not relevant.

Teasley, S. (1995). The role of talk in children's peer collaborations. *Developmental Psychology*, 31(2), 207-220. This study looked at the role of talk in peer collaboration around generating hypotheses in a computer generated scientific reasoning task. They compared talk in different situations with the hypothesis produced in each. As expected from previous research "The benefits for talk were more pronounced when children talked with a partner than when they talked with themselves," or when they worked with partners and did not talk. "...[T]talk dyads produced more talk overall and more interpretive types of talk than talk alones."